News-Shock Subroutine for “Prof. Uhlig’s Toolkit” *

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Abstract

The roles of “news shocks” in dynamic general equilibrium models have been highlighted since Beaudry and Portier (2004). In this paper, we explain how to use “news-shock subroutine for Prof. Uhlig’s toolkit.” This subroutine can make to calculate the responses to news-shock easily. Users of the toolkit by Professor Harald Uhlig can do the news-shock experiments by this subroutine.

1 Introduction

The roles of news about future, “news shocks,” in dynamic general equilibrium models (DGE models, hereafter) have been highlighted since Beaudry and Portier (2004). Especially, economists have made many efforts to construct models that can generate *the Pigou cycles*:[1] the news about future causes the boom at the current period.[2] It is well known that the simple RBC model cannot generate the Pigou cycles.

In this paper, we explain how to use “news-shock subroutine for Prof. Uhlig’s toolkit.” This subroutine can make to calculate the responses to news-shock easily. The toolkit by Professor Harald Uhlig (Prof. Uhlig’s, toolkit hereafter) is often used to solve the non-linear dynamic general equilibrium model by the linear approximation. Users of Prof. Uhlig’s toolkit[3] can do the news-shock experiments by this subroutine.

The organization of this recipe is as follows. In the next section, we describe the files contained in “news_shock_kit.zip,” and explain how to use it briefly. In Section 3, we describe the analytical explanation for the calculation of the responses to news shocks. The application

*I would like to thank Tomoyuki Nakajima, Keiichiro Kobayashi, and Masaru Inaba and for helpful comments. I would also thank to Harald Uhlig and members of the toolkit news group for the development of an excellent toolkit. The latest version of this file and the subroutine file are downlodable from my website: “http://kengo.nutahara.googlepages.com/”

[1] Some economists call this phenomenon various names: the news-driven cycle, the expectations-driven cycle, the boom-bust cycle, and so on.

[2] As recent papers, see Christiano, Motto and Rostagno (2006), Jaimovich and Rebelo (2006), Kobayashi, Nakajima and Inaba (2007), Kobayashi and Nutahara (2007), and so on.

[3] Prof. Uhlig’s toolkit is downloadable from “http://www2.wiwi.hu-berlin.de/institute/wpol/html/toolkit.htm.”

to simple RBC model is shown in Section 4. Section 5 applies our toolkit to the model of Kobayashi and Nutahara (2007) which can produce Pigou cycle; the news about future causes a boom at the current period.

2 How to Use

The files contained the zip file "news_shock_kit.zip" are followings:

1. "do_news_shock.m": the subroutine file for the news shock experiments. This file calculate the responses to news shocks, and draw graphs.
2. "manual_news.pdf": you are reading it now!
3. "example1-simpleRBC"(folder): the application of do_news_shock.m to the simple RBC model.
4. "example2-Kobayashi-Nutahara2007"(folder): the application of do_news_shock.m to the model of Kobayashi and Nutahara (2007).

It is quite easy to use the news-shock subroutine. Users of "Prof. Uhlig's toolkit" can do news-shock experiments just replacing "do_it" by "do_news_shock" in their matlab codes! Generally, user should do the followings:

1. Define the matrices $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{D}$, $\boldsymbol{F}$, $\boldsymbol{G}$, $\boldsymbol{H}$, $\boldsymbol{J}$, $\boldsymbol{K}$, $\boldsymbol{L}$, $\boldsymbol{M}$, and $\boldsymbol{N}$ for (1), (2) and (3).
2. Assign names of variables to VARNAMES, and GNP_INDEX.
3. Set PERIOD. It is 1, 4, and 12 if the model is specified to annual, quarterly, and monthly one, respectively.
4. (If you want to change) Set options: NEWS_LAG, NEWS_SIM_HORIZON, and NEWS_NUM.
5. Write "do_news_shock" in your matlab code!

The followings are options for news-shock subroutine;

1. NEWS_LAG: after "NEWS_LAG" periods, the news realizes (default; 1 year)
2. NEWS_SIM_HORIZON : horizon (periods) for news shock experiments in the graph (default; 5 year)
3. NEWS_NUM : news is about "NEWS_NUM"-th exogenous variable (default; 1)

Specifying these options, users can do many types of news-shock experiments.

Some users might have motivations to produce the original figure from the results of this subroutine. The main output variables are as follows.

- xtil_1: simulated endogenous variables if the news turns to be false. (The rows mean variables, and the columns mean periods. The order of the variables is the same as in VARNAMES.)
- xtil_2: simulated endogenous variables if the news is correct.

- ztil_2: simulated exogenous variables if the news is correct.
- qtr: horizontal axis for the graph.

3 Theory for News-Shock Experiments

Here, we explain how to compute the responses to news shocks. Our strategy is as follows.

1. Use the policy functions for the model *without news shocks*.
2. Use the equilibrium system, calculate the (variant) policy functions with news shocks for each periods.

This process is the same as in "do_news_shock.m".

3.1 Linearized System and Policy Functions without News Shocks

First, we employ the log-linear approximation to approximate the de-trended equilibrium system. Following Uhlig (1999), the matrix representation of the linearized equilibrium system without news shocks is

$$\boldsymbol{A}\boldsymbol{x}_t + \boldsymbol{B}\boldsymbol{x}_{t-1} + \boldsymbol{C}\boldsymbol{y}_t + \boldsymbol{D}\boldsymbol{z}_t = \boldsymbol{0}, \tag{1}$$

$$\mathbb{E}_t\Big[\boldsymbol{F}\boldsymbol{x}_{t+1} + \boldsymbol{G}\boldsymbol{x}_t + \boldsymbol{H}\boldsymbol{x}_{t-1} + \boldsymbol{J}\boldsymbol{y}_{t+1} + \boldsymbol{K}\boldsymbol{y}_t + \boldsymbol{L}\boldsymbol{z}_{t+1} + \boldsymbol{M}\boldsymbol{z}_t\Big] = \boldsymbol{0}, \tag{2}$$

$$\boldsymbol{z}_{t+1} = \boldsymbol{N}\boldsymbol{z}_t + \boldsymbol{\varepsilon}_{t+1}, \quad \mathbb{E}_t\Big[\boldsymbol{\varepsilon}_{t+1}\Big] = \boldsymbol{0}, \tag{3}$$

where $\boldsymbol{x}_t$, $\boldsymbol{y}_t$, and $\boldsymbol{z}_t$ denote vectors of endogenous state variables, endogenous jump variables, and exogenous variables, respectively. By the method of Uhlig (1999), we obtain the policy functions;

$$\boldsymbol{x}_t = \boldsymbol{P}\boldsymbol{x}_{t-1} + \boldsymbol{Q}\boldsymbol{z}_t, \tag{4}$$

$$\boldsymbol{y}_t = \boldsymbol{R}\boldsymbol{x}_{t-1} + \boldsymbol{S}\boldsymbol{z}_t. \tag{5}$$

For our news-shock experiments, we introduce the more simple form of the equilibrium system and the policy functions. (1) and (2) can be summarized as follows:

$$\mathbb{E}_t\Big[\tilde{\boldsymbol{F}}\tilde{\boldsymbol{x}}_{t+1} + \tilde{\boldsymbol{G}}\tilde{\boldsymbol{x}}_t + \tilde{\boldsymbol{H}}\tilde{\boldsymbol{x}}_{t-1} + \tilde{\boldsymbol{L}}\boldsymbol{z}_{t+1} + \tilde{\boldsymbol{M}}\boldsymbol{z}_t\Big] = \boldsymbol{0}, \tag{6}$$

where

$$\tilde{\boldsymbol{x}}_t = \begin{bmatrix} \boldsymbol{x}_t \\ \boldsymbol{y}_t \end{bmatrix}, \quad \tilde{\boldsymbol{F}} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{F} & \boldsymbol{J} \end{bmatrix}, \quad \tilde{\boldsymbol{G}} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{C} \\ \boldsymbol{G} & \boldsymbol{K} \end{bmatrix}, \quad \tilde{\boldsymbol{L}} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{L} \end{bmatrix}, \quad \tilde{\boldsymbol{M}} = \begin{bmatrix} \boldsymbol{D} \\ \boldsymbol{M} \end{bmatrix}. \tag{7}$$

Similar to this, (4) and (5) can be summarized as follows:

$$\tilde{\boldsymbol{x}}_t = \tilde{\boldsymbol{P}}\tilde{\boldsymbol{x}}_{t-1} + \tilde{\boldsymbol{Q}}\boldsymbol{z}_t, \tag{8}$$

where

$$\tilde{\boldsymbol{x}}_t = \begin{bmatrix} \boldsymbol{x}_t \\ \boldsymbol{y}_t \end{bmatrix}, \qquad \tilde{\boldsymbol{P}} = \begin{bmatrix} \boldsymbol{P} & \boldsymbol{0} \\ \boldsymbol{R} & \boldsymbol{0} \end{bmatrix}, \qquad \tilde{\boldsymbol{Q}} = \begin{bmatrix} \boldsymbol{Q} \\ \boldsymbol{S} \end{bmatrix}, \tag{9}$$

3.2 Policy Functions with News Shocks

Our news-shock experiment is the same as Christiano, Motto and Rostagno (2005). The procedure is the followings;

1. For $t < T_a$, the economy is at the steady-state.
2. At $t = T_a$, news arrives; $\boldsymbol{z}_{T_b} = \overline{\boldsymbol{z}} \neq \boldsymbol{0}$ occurs at T_b.
3. At $t = T_b$, agents know that news is correct or not.

The important point is that the policy functions with news shocks are variant.

For $t = T_b$, since there is no news shock, the policy functions are

$$\tilde{\boldsymbol{x}}_{T_b} = \tilde{\boldsymbol{P}}\tilde{\boldsymbol{x}}_{T_b-1} + \tilde{\boldsymbol{Q}}\boldsymbol{z}_{T_b}. \tag{10}$$

We obtain the policy functions in a backward way using (10) and (6). At $t = T_b - 1$, (6) becomes

$$\begin{aligned} &\mathbb{E}_{T_b-1}\Big[\tilde{\boldsymbol{F}}\tilde{\boldsymbol{x}}_{T_b} + \tilde{\boldsymbol{G}}\tilde{\boldsymbol{x}}_{T_b-1} + \tilde{\boldsymbol{H}}\tilde{\boldsymbol{x}}_{T_b-2} + \tilde{\boldsymbol{L}}\boldsymbol{z}_{T_b} + \tilde{\boldsymbol{M}}\boldsymbol{z}_{T_b-1}\Big] = \boldsymbol{0}, \\ \iff\quad &\tilde{\boldsymbol{F}}\Big[\tilde{\boldsymbol{P}}\tilde{\boldsymbol{x}}_{T_b-1} + \tilde{\boldsymbol{Q}}\overline{\boldsymbol{z}}\Big] + \tilde{\boldsymbol{G}}\tilde{\boldsymbol{x}}_{T_b-1} + \tilde{\boldsymbol{H}}\tilde{\boldsymbol{x}}_{T_b-2} + \tilde{\boldsymbol{L}}\overline{\boldsymbol{z}} = \boldsymbol{0}. \end{aligned} \tag{11}$$

Then, the policy functions for $t = T_b - 1$ are

$$\tilde{\boldsymbol{x}}_{T_b-1} = \boldsymbol{W}_{T_b-1}\tilde{\boldsymbol{x}}_{Tb-2} + \boldsymbol{V}_{T_b-1}\overline{\boldsymbol{z}}. \tag{12}$$

where

$$\boldsymbol{W}_{T_b-1} = -\Big[\tilde{\boldsymbol{F}}\tilde{\boldsymbol{P}} + \tilde{\boldsymbol{G}}\Big]^{-1}\tilde{\boldsymbol{H}}, \tag{13}$$

$$\boldsymbol{V}_{T_b-1} = -\Big[\tilde{\boldsymbol{F}}\tilde{\boldsymbol{P}} + \tilde{\boldsymbol{G}}\Big]^{-1}\Big[\tilde{\boldsymbol{F}}\tilde{\boldsymbol{Q}} + \tilde{\boldsymbol{L}}\Big]. \tag{14}$$

For $t = T_b - 2$, (6) is

$$\begin{aligned} &\mathbb{E}_t\Big[\tilde{\boldsymbol{F}}\tilde{\boldsymbol{x}}_{t+1} + \tilde{\boldsymbol{G}}\tilde{\boldsymbol{x}}_t + \tilde{\boldsymbol{H}}\tilde{\boldsymbol{x}}_{t-1} + \tilde{\boldsymbol{L}}\tilde{\boldsymbol{z}}_{t+1} + \tilde{\boldsymbol{M}}\tilde{\boldsymbol{z}}_t\Big] = \boldsymbol{0}, \\ \iff\quad &\tilde{\boldsymbol{F}}\Big[\boldsymbol{W}_{t+1}\tilde{\boldsymbol{x}}_t + \boldsymbol{V}_{t+1}\overline{\boldsymbol{z}}\Big] + \tilde{\boldsymbol{G}}\tilde{\boldsymbol{x}}_t + \tilde{\boldsymbol{H}}\tilde{\boldsymbol{x}}_{t-1} = \boldsymbol{0}. \end{aligned} \tag{15}$$

Thus, the policy functions for $t = T_b - 2$ are computed as follows;

$$\tilde{\boldsymbol{x}}_t = \boldsymbol{W}_t\tilde{\boldsymbol{x}}_{t-1} + \boldsymbol{V}_t\overline{\boldsymbol{z}}, \tag{16}$$

where

$$\boldsymbol{W}_t = -\Big[\tilde{\boldsymbol{F}}\boldsymbol{W}_{t+1} + \tilde{\boldsymbol{G}}\Big]^{-1}\tilde{\boldsymbol{H}}, \tag{17}$$

$$\boldsymbol{V}_t = -\Big[\tilde{\boldsymbol{F}}\boldsymbol{W}_{t+1} + \tilde{\boldsymbol{G}}\Big]^{-1}\tilde{\boldsymbol{F}}\boldsymbol{V}_{t+1}. \tag{18}$$

In the same logic, the policy functions for $T_a \leq t \leq T_b - 3$ are the same as (16) - (18).

4 Application I: Simple RBC Model

Here, we explain how to use our toolkit through the application to the simple RBC model.

4.1 Definition of Variables

The followings are the variables in the model.

- c_t: consumption
- n_t: labor
- k_t: capital stock at the end of period
- i_t: investment
- w_t: wage rate
- r_t: rental rate of capital
- ζ_t: Harrod neutral technology (trend stationary)
- g_t: growth rate of ζ_t ($g_t := \log(\zeta_t/\zeta_{t-1})$)
- y_t: output

4.2 The Economy

Households:

$$\max_{c_t,n_t,i_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{[c_t^{1-\xi}(1-n_t)^{\xi}]^{1-\varepsilon}}{1-\varepsilon}, \tag{19}$$

$$\text{s.t. } c_t + i_t \leq w_t n_t + r_t k_t, \tag{20}$$

$$k_t = (1-\delta)k_{t-1} + i_t. \tag{21}$$

Firms:

$$\max_{k_t,n_t} y_t - r_t k_t - w_t n_t, \tag{22}$$

$$\text{s.t. } y_t = k_t^{\alpha}\Big[\zeta_t n_t\Big]^{1-\alpha}. \tag{23}$$

Market Clearing Conditions:

$$c_t + +i_t = y_t. \tag{24}$$

Exogenous Technology:

$$\log(\zeta_{t+1}/\zeta_t) = \rho_g \log(\zeta_t/\zeta_{t-1}) + (1-\rho_g)\log(\overline{g}) + \varepsilon_t^g, \tag{25}$$

where ρ_g is less than one.

4.3 Equilibrium System

Given the evolution of the exogenous variables, the equilibrium system is the following:

$$(1-\xi)c_t^{(1-\xi)(1-\epsilon)-1}(1-n_t)^{\xi(1-\epsilon)} = \lambda_t, \tag{26}$$
$$\frac{\xi}{1-\xi} \cdot \frac{c_t}{1-n_t} = w_t, \tag{27}$$
$$1 = \beta\, \mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left(r_{t+1} + 1 - \delta\right)\right], \tag{28}$$
$$y_t = k_{t-1}^{\alpha}\left[\zeta_t n_t\right]^{1-\alpha}, \tag{29}$$
$$r_t = \alpha \cdot \frac{y_t}{k_{t-1}}, \tag{30}$$
$$w_t = (1-\alpha) \cdot \frac{y_t}{n_t}, \tag{31}$$
$$c_t + i_t = y_t, \tag{32}$$
$$k_t = (1-\delta)k_{t-1} + i_t. \tag{33}$$

There are 7 equations and 7 variables:

$$c_t,\ n_t,\ k_t,\ i_t,\ y_t,\ w_t,\ r_t.$$

For the de-trended system, we introduce the following notation:

$$\tilde{a}_t := \frac{a_t}{\zeta_t}, \tag{34}$$

for $a_t = c_t$, k_t, i_t, w_t, and y_t.

The de-trended system is

$$(1-\xi)\tilde{c}_t^{(1-\xi)(1-\varepsilon)-1}(1-n_t)^{\xi(1-\varepsilon)} = \tilde{\lambda}_t, \tag{35}$$
$$\frac{\xi}{1-\xi} \cdot \frac{\tilde{c}_t}{1-n_t} = \tilde{w}_t, \tag{36}$$
$$1 = \beta\, \mathbb{E}_t\left[\frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t}(1+g_{t+1})^{(1-\xi)(1-\varepsilon)-1}\left(r_{t+1} + 1 - \delta\right)\right], \tag{37}$$
$$\tilde{y}_t = \left[\frac{\tilde{k}_{t-1}}{1+g_t}\right]^{\alpha} n_t^{1-\alpha}, \tag{38}$$
$$r_t = \alpha \cdot \frac{\tilde{y}_t}{\tilde{k}_{t-1}}(1+g_t), \tag{39}$$
$$\tilde{w}_t = (1-\alpha) \cdot \frac{\tilde{y}_t}{n_t}, \tag{40}$$
$$\tilde{c}_t + \tilde{i}_t = \tilde{y}_t, \tag{41}$$
$$\tilde{k}_t = \frac{1-\delta}{1+g_t}\tilde{k}_{t-1} + \tilde{i}_t. \tag{42}$$

There are 7 equations and 7 variables:

$$\tilde{c}_t,\ n_t,\ \tilde{k}_t,\ \tilde{i}_t,\ \tilde{y}_t,\ \tilde{w}_t,\ r_t.$$

4.4 News-Shock Experiments

To solve the policy function, we divide the equilibrium system into two parts:

- Intra-temporal Conditions (1): (35), (36), (38), (39), (40), (41), and (42).
- Inter-temporal Conditions (2): (37).

Variables are $\boldsymbol{x} := [\tilde{k}_t]$, $\boldsymbol{y}_t := [\tilde{c}_t, n_t, \tilde{i}_t, \tilde{y}_t, r_t, \tilde{w}_t]'$, and $\boldsymbol{z} := [g_t]$. The matlab file "simple_RBC_news.m" takes the log-linear approximation of the model.[4] Figure 1 shows the responses to news shock about one percent deviation after one year, and it turns out to be false after one year.[5] It is easily check that the simple RBC model cannot produce the Pigou cycle; labor, investment and output fall when the news arrives.

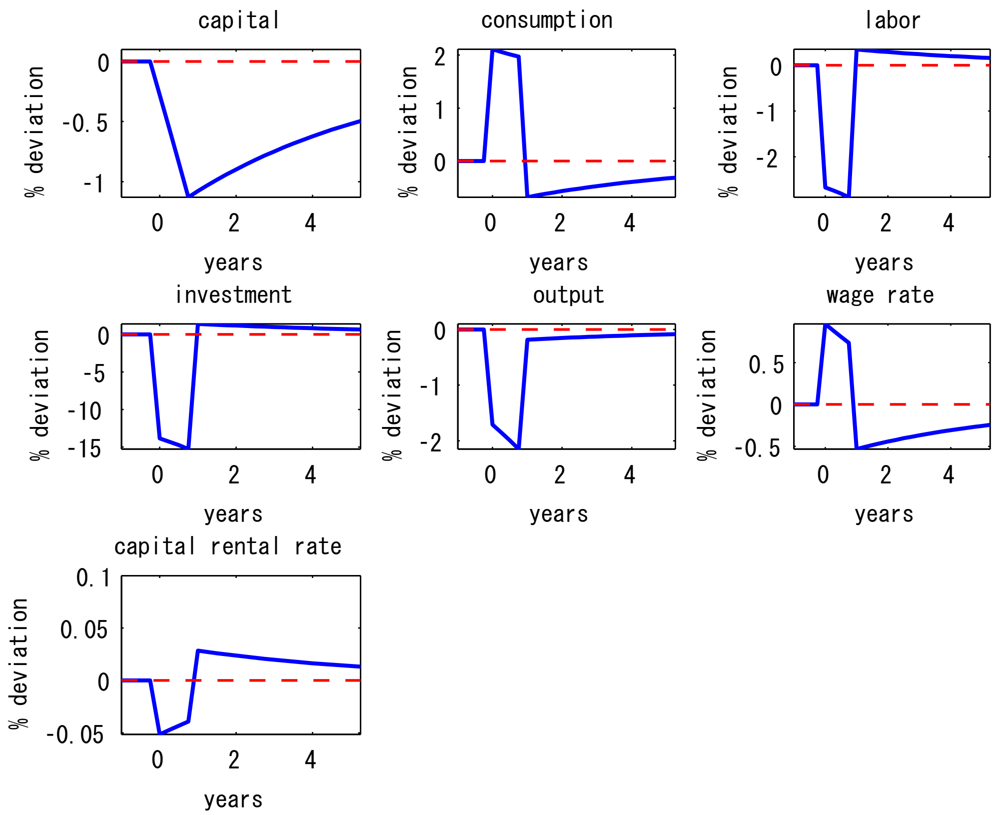

Figure 1: News-Shock Experiments (1): simple RBC (not realized)

[4]To run the program, users have to install the symbolic math toolbox for matlab.
[5]News-shock subroutine draw two figure: both the news realizes and not.

5 Application II: Kobayashi and Nutahara (2007)

In this section, we explain the model of Kobayashi and Nutahara (2007). The features of this model are followings:

1. input cost is subject to collateral constraint because of the lack of commitment problem. (capital is collateralized.)
2. adjustment cost of capital ($\Phi(i_t/k_t)$ where $\Phi(0) = 0$, $\Phi(\delta) = \delta$, $\Phi'(x) > 0$, and $\Phi''(x) < 0$)

Through this friction, the model can produce *the Pigou cycle*: the news about future technology growth causes the boom at the current period.[6]

5.1 Definition of Variables

The followings are the variables in this model.[7]

- c_t: consumption
- n_t: labor supply of worker
- $\tilde{n}_t$: labor demand of manager
- k_t: capital stock at the end of period
- i_t: input of investment
- z_t: output of investment
- w_t: wage rate
- π_t: profit of manager
- A_t: Hicks neutral technology (log level stationary)
- ζ_t: Harrod neutral technology (trend stationary)
- g_t: growth rate of ζ_t ($g_t := \log(\zeta_t/\zeta_{t-1})$)
- m_t: material input
- θ_t: gross output
- y_t: value added
- q_t: shadow price of capital (Tobin's Q)

[6] This model is one of two models in Kobayashi and Nutahara (2007). Two-agent version of this model is also explained in Koabayashi and Nutahara (2007), and it can produce the recession if the news turns to be false.

[7] Some notations are different from the paper.

5.2 The Economy

(Consolidated) Households: pair of worker and manager

$$\max_{c_t,n_t,k_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[c_t^{1-\gamma}(1-n_t)^{\gamma}\right]^{1-\varepsilon}}{1-\varepsilon}, \tag{43}$$

$$\text{s.t. } c_t + i_t \leq w_t n_t + \pi_t, \tag{44}$$

$$\pi_t := \theta_t - w_t n_t - m_t, \tag{45}$$

$$\theta_t := A_t \cdot m_t^{\eta} \cdot k_{t-1}^{(1-\eta)\alpha} \cdot \left[\zeta_t n_t\right]^{(1-\eta)(1-\alpha)}, \tag{46}$$

$$w_t n_t + m_t \leq \varphi q_t k_{t-1}, \tag{47}$$

$$k_t = (1-\delta)k_{t-1} + z_t, \tag{48}$$

$$z_t = \Phi\left(\frac{i_t}{k_{t-1}}\right)k_{t-1}. \tag{49}$$

Market Clearing Conditions:

$$c_t + c_t' + i_t = y_t, \tag{50}$$

$$y_t = \theta_t - m_t. \tag{51}$$

Exogenous Technology:

$$\log(z_{t+1}) = \rho_z \log(z_{t+1}) + (1-\rho_z)\log(\bar{z}) + \varepsilon_t^z, \tag{52}$$

$$\log(\zeta_{t+1}/\zeta_t) = \rho_g \log(\zeta_t/\zeta_{t-1}) + (1-\rho_g)\log(\bar{g}) + \varepsilon_t^g, \tag{53}$$

where ρ_z and ρ_g are less than one.

5.3 Equilibrium System

Given the evolution exogenous variables, the equilibrium system is

$$\frac{\gamma}{1-\gamma}\cdot\frac{c_t}{1-n_t}=\frac{(1-\eta)(1-\alpha)}{1+x_t}\cdot\frac{\theta_t}{n_t}, \tag{54}$$

$$q_t=\beta\mathbb{E}_t\Bigg[\frac{c_{t+1}^{(1-\gamma)(1-\varepsilon)-1}(1-n_{t+1})^{\gamma(1-\varepsilon)}}{c_t^{(1-\gamma)(1-\varepsilon)-1}(1-n_t)^{\gamma(1-\varepsilon)}}\Bigg\{(1-\delta+\varphi x_t)q_{t+1}+(1-\eta)\alpha\cdot\frac{\theta_t}{k_{t-1}} + q_{t+1}\Bigg[\Phi\Big(\frac{i_t}{k_{t-1}}\Big)-\Phi'\Big(\frac{i_t}{k_{t-1}}\Big)\frac{i_t}{k_{t-1}}\Bigg]\Bigg\}\Bigg], \tag{55}$$

$$\theta_t=z_t\cdot m_t^{\eta}\cdot k_{t-1}^{(1-\eta)\alpha}\cdot\Big[\zeta_t n_t\Big]^{(1-\eta)(1-\alpha)}, \tag{56}$$

$$\frac{\gamma}{1-\gamma}\cdot\frac{c_t}{1-n_t}n_t+m_t=\varphi q_t k_{t-1}, \tag{57}$$

$$1=\eta\cdot\frac{\theta_t}{m_t}-x_t, \tag{58}$$

$$q_t=\Bigg[\Phi'\Big(\frac{i_t}{k_{t-1}}\Big)\Bigg]^{-1}, \tag{59}$$

$$k_t=(1-\delta)k_{t-1}+\Phi\Big(\frac{i_t}{k_{t-1}}\Big)k_{t-1}, \tag{60}$$

$$c_t+i_t=y_t, \tag{61}$$

$$y_t=\theta_t-m_t. \tag{62}$$

There are 9 equations and 9 variables:

$$c_t,\ n_t,\ q_t,\ k_t,\ i_t,\ m_t,\ y_t,\ \theta_t,\ x_t.$$

Note that q_t equals to the ration of the Lagrange multiplier of the evolution of capital evolution to that of budget constraint. x_t denote the power of collateral constraint, which is the ratio of the Lagrange multipliers of collateral constraint to that of budget constraint.

For the de-trended system, we introduce the following notation:

$$\tilde{a}_t:=\frac{a_t}{\zeta_t}, \tag{63}$$

for $a_t=c_t, k_t, i_t, m_t, y_t$, and θ_t.

The de-trended system is

$$\frac{\gamma}{1-\gamma}\cdot\frac{\tilde{c}_t}{1-n_t}=\frac{(1-\eta)(1-\alpha)}{1+x_t}\cdot\frac{\tilde{\theta}_t}{n_t}, \tag{64}$$

$$\begin{aligned} q_t = \beta\mathbb{E}_t\Bigg[& \frac{\tilde{c}_{t+1}^{(1-\gamma)(1-\varepsilon)-1}(1-n_{t+1})^{\gamma(1-\varepsilon)}}{\tilde{c}_t^{(1-\gamma)(1-\varepsilon)-1}(1-n_t)^{\gamma(1-\varepsilon)}}(1+g_{t+1})^{(1-\gamma)(1-\varepsilon)-1} \\ & \Big\{(1-\delta+\varphi x_t)q_{t+1}+(1-\eta)\alpha\cdot\frac{\tilde{\theta}_t}{\tilde{k}_{t-1}}(1+g_t) \\ & + q_{t+1}\Big[\Phi\Big(\frac{\tilde{i}_t}{\tilde{k}_{t-1}}(1+g_t)\Big)-\Phi'\Big(\frac{\tilde{i}_t}{\tilde{k}_{t-1}}(1+g_t)\Big)\frac{\tilde{i}_t}{\tilde{k}_{t-1}}(1+g_t)\Big]\Big\}\Bigg], \end{aligned} \tag{65}$$

$$\tilde{\theta}_t = z_t\cdot\tilde{m}_t^{\eta}\cdot\left[\frac{\tilde{k}_{t-1}}{1+g_t}\right]^{(1-\eta)\alpha}\cdot n_t^{(1-\eta)(1-\alpha)}, \tag{66}$$

$$\frac{(1-\eta)(1-\alpha)}{1+x_t}\cdot\frac{\tilde{\theta}_t}{n_t}n_t+\tilde{m}_t=\varphi q_t\frac{\tilde{k}_{t-1}}{1+g_t}, \tag{67}$$

$$1=\eta\cdot\frac{\tilde{\theta}_t}{\tilde{m}_t}-x_t, \tag{68}$$

$$q_t=\left[\Phi'\Big(\frac{\tilde{i}_t}{\tilde{k}_{t-1}}(1+g_t)\Big)\right]^{-1}, \tag{69}$$

$$\tilde{k}_t=\frac{1-\delta}{1+g_t}\tilde{k}_{t-1}+\Phi\Big(\frac{\tilde{i}_t}{\tilde{k}_{t-1}}(1+g_t)\Big)\frac{\tilde{k}_{t-1}}{1+g_t}, \tag{70}$$

$$\tilde{c}_t+\tilde{i}_t=\tilde{y}_t, \tag{71}$$

$$\tilde{y}_t=\tilde{\theta}_t-\tilde{m}_t. \tag{72}$$

There are 9 equations and 9 variables:

$$\tilde{c}_t,\ n_t,\ q_t,\ \tilde{k}_t,\ \tilde{i}_t,\ \tilde{m}_t,\ \tilde{y}_t,\ \tilde{\theta}_t,\ x_t.$$

5.4 News-Shock Experiments

To solve the policy function, we divide the equilibrium system into two parts:

- Intra-temporal Conditions (1): (64), (66), (67), (68), (69), (70), (71), and (72).
- Inter-temporal Conditions (2): (65).

Variables are $\boldsymbol{x} := [\tilde{k}_t]$, $\boldsymbol{y}_t := [\tilde{c}_t, n_t, \tilde{i}_t, x_t, \tilde{\theta}_t, \tilde{y}_t, \tilde{m}_t, q_t]'$, and $\boldsymbol{z} := [g_t, z_t]$. The matlab file "KN2007.m" takes the log-linear approximation of the model.[8] Figure 2 shows the responses to news shock about one percent deviation of technology growth after one year, and it turns out to be false after one year.[9] It is easily check that *this model can produce the Pigou cycle*; consumption, labor, investment and output increase when the news arrives! Furthermore, this model can produce the procyclical movement of Tobin's Q.[10]

[8] To run the program, users have to install the symbolic math toolbox for matlab.

[9] News-shock subroutine draw two figure: both the news realizes and not.

[10] Many other models for the Pigou cycle cannot produce the procyclical movement of Tobin's Q. The model with collateral constraints like Kobayashi and Nutahara (2007) has an advantage in this sense.

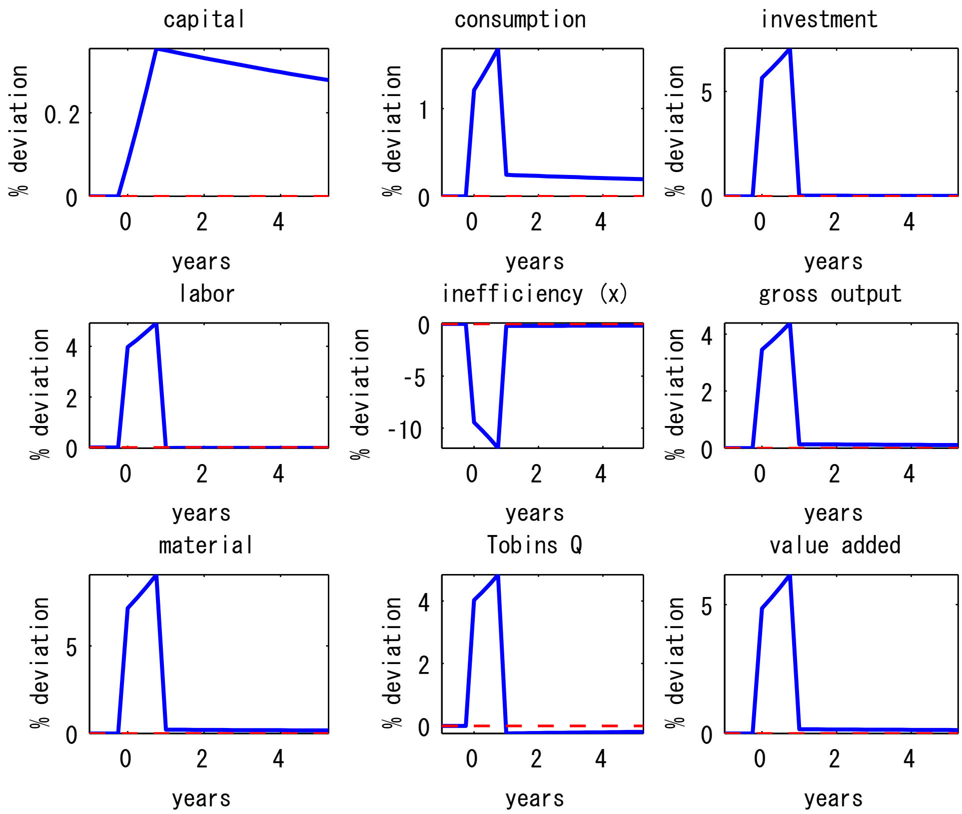

Figure 2: News-Shock Experiments (2): Kobayashi and Nutahara (2007) (not realized)

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